Design of Isolated R.C. Footings

1. General

Most of the structures built by us are made of reinforced concrete. Here, the part of the structure above ground level is called as the superstructure, where the part of the structure below the ground level is called as the substructure. Footings are located below the ground level and are also referred as foundations. Foundation is that part of the structure which is in direct contact with soil. The R.C. structures consist of various structural components which act together to resist the applied loads and transfer them safely to soil. In general the loads applied on slabs in buildings are transferred to soil through beams, columns and footings. Footings are that part of the structure which are generally located below ground Level. They are also referred as foundations. Footings transfer the vertical loads, Horizontal loads, Moments, and other forces to the soil.

The important purpose of foundation are as follows;

1. To transfer forces from superstructure to firm soil below.
2. To distribute stresses evenly on foundation soil such that foundation soil neither fails nor experiences excessive settlement.
3. To develop an anchor for stability against overturning.
4. To provide an even surface for smooth construction of superstructure.

Due to the loads and soil pressure, footings develop Bending moments and Shear forces. Calculations are made as per the guidelines suggested in IS 456 2000 to resist the internal forces.

2. Types of Foundations

Based on the position with respect to ground level, Footings are classified into two types;

1. Shallow Foundations
2. Deep Foundations

Shallow Foundations are provided when adequate SBC is available at relatively short depth below ground level. Here, the ratio of $D_f / B < 1$, where $D_f$ is the depth of footing and $B$ is the width of footing. Deep Foundations are provided when adequate SBC is available at large depth below ground level. Here the ratio of $D_f / B >= 1$.

2.1 Types of Shallow Foundations

The different types of shallow foundations are as follows:

- Isolated Footing
- Combined footing
- Strap Footing
- Strip Footing
- Mat/Raft Foundation
- Wall footing
Some of the popular types of shallow foundations are briefly discussed below.

a) Isolated Column Footing

These are independent footings which are provided for each column. This type of footing is chosen when

- SBC is generally high
- Columns are far apart
- Loads on footings are less

The isolated footings can have different shapes in plan. Generally it depends on the shape of column cross section. Some of the popular shapes of footings are;

- Square
- Rectangular
- Circular

The isolated footings essentially consists of bottom slab. These bottom slabs can be either flat, stepped or sloping in nature. The bottom of the slab is reinforced with steel mesh to resist the two internal forces namely bending moment and shear force.

The sketch of a typical isolated footing is shown in Fig. 1.

b) Combined Column Footing

These are common footings which support the loads from 2 or more columns. Combined footings are provided when

- SBC is generally less
- Columns are closely spaced
- Footings are heavily loaded

In the above situations, the area required to provide isolated footings for the columns generally overlap. Hence, it is advantageous to provide single combined footing. In some cases the columns are located on or close to property line. In such cases footings cannot be extended on one side. Here, the footings of exterior and interior columns are connected by the combined footing.
Combined footings essentially consist of a common slab for the columns it is supporting. These slabs are generally rectangular in plan. Sometimes they can also be trapezoidal in plan (refer Fig. 2). Combined footings can also have a connecting beam and a slab arrangement, which is similar to an inverted T–beam slab.

c) **Strap Footing**

An alternate way of providing combined footing located close to property line is the strap footing. In strap footing, independent slabs below columns are provided which are then connected by a strap beam. The strap beam does not remain in contact with the soil and does not transfer any pressure to the soil. Generally it is used to combine the footing of the outer column to the adjacent one so that the footing does not extend in the adjoining property. A typical strap footing is shown in Fig. 3.
d) **Strip Footing**

Strip footing is a continuous footing provided under columns or walls. A typical strip footing for columns is shown in Fig. 4.

![Fig. 4 Plan and section of typical strip footing](image)

e) **Mat Foundation**

Mat foundation covers the whole plan area of structure. The detailing is similar to two way reinforced solid floor slabs or flat slabs. It is a combined footing that covers the entire area beneath a structure and supports all the walls and columns. It is normally provided when

- Soil pressure is low
- Loads are very heavy
- Spread footings cover > 50% area

A typical mat foundation is shown in Fig. 5.

![Fig. 5 Plan and section of typical strip footing](image)

### 2.2 Types of Deep Foundations

Deep foundations are provided when adequate SBC is available at large depth below GL. There are different types of deep foundations. Some of the common types of deep foundations are listed below.

- Pile Foundation
- Pier Foundation
- Well Foundation
3. **Bearing Capacity of Soil**

The safe bearing capacity of soil is the safe extra load soil can withstand without experiencing shear failure. The Safe Bearing Capacity (SBC) is considered unique at a particular site. But it also depends on the following factors:

- Size of footing
- Shape of footing
- Inclination of footing
- Inclination of ground
- Type of load
- Depth of footing etc.

SBC alone is not sufficient for design. The allowable bearing capacity is taken as the smaller of the following two criteria:

- Limit states of shear failure criteria (SBC)
- Limit states of settlement criteria

Based on ultimate capacity, i.e., shear failure criteria, the SBC is calculated as

\[
SBC = \frac{\text{Total load}}{\text{Area of footing}}
\]

Usually the Allowable Bearing Pressure (ABP) varies in the range of 100 kN/m\(^2\) to 400 kN/m\(^2\). The area of the footing should be so arrived that the pressure distribution below the footing should be less than the allowable bearing pressure of the soil. Even for symmetrical Loading, the pressure distribution below the footing may not be uniform. It depends on the Rigidity of footing, Soil type and Conditions of soil. In case of Cohesive Soil and Cohesion less Soil the pressure distribution varies in a nonlinear way. However, while designing the footings a linear variation of pressure distribution from one edge of the footing to the other edge is assumed. Once the pressure distribution is known, the bending moment and shear force can be determined and the footing can be designed to safely resist these forces.

4. **Design of Isolated Column Footing**

The objective of design is to determine:

- Area of footing
- Thickness of footing
- Reinforcement details of footing (satisfying moment and shear considerations)
- Check for bearing stresses and development length

This is carried out considering the loads of footing, SBC of soil, Grade of concrete and Grade of steel. The method of design is similar to the design of beams and slabs. Since footings are buried, deflection control is not important. However, crack widths should be less than 0.3 mm.

The steps followed in the design of footings are generally iterative. The important steps in the design of footings are:

- Find the area of footing (due to service loads)
- Assume a suitable thickness of footing
• Identify critical sections for flexure and shear
• Find the bending moment and shear forces at these critical sections (due to factored loads)
• Check the adequacy of the assumed thickness
• Find the reinforcement details
• Check for development length
• Check for bearing stresses

Limit state of collapse is adopted in the design of isolated column footings. The various design steps considered are:

• Design for flexure
• Design for shear (one way shear and two way shear)
• Design for bearing
• Design for development length

The materials used in RC footings are concrete and steel. The minimum grade of concrete to be used for footings is M20, which can be increased when the footings are placed in aggressive environment, or to resist higher stresses.

Cover: The minimum thickness of cover to main reinforcement shall not be less than 50 mm for surfaces in contact with earth face and not less than 40 mm for external exposed face. However, where the concrete is in direct contact with the soil the cover should be 75 mm. In case of raft foundation the cover for reinforcement shall not be less than 75 mm.

Minimum reinforcement and bar diameter: The minimum reinforcement according to slab and beam elements as appropriate should be followed, unless otherwise specified. The diameter of main reinforcing bars shall not be less 10 mm. The grade of steel used is either Fe 415 or Fe 500.

5. Specifications for Design of footings as per IS 456 : 2000

The important guidelines given in IS 456 : 2000 for the design of isolated footings are as follows:

34.1 General

Footings shall be designed to sustain the applied loads, moments and forces and the induced reactions and to ensure that any settlement which may occur shall be as nearly uniform as possible, and the safe bearing capacity of the soil is not exceeded (see IS 1904).

34.1.1 In sloped or stepped footings the effective cross-section in compression shall be limited by the area above the neutral plane, and the angle of slope or depth and location of steps shall be such that the design requirements are satisfied at every section. Sloped and stepped footings that are designed as a unit shall be constructed to assure action as a unit.

34.1.2 Thickness at the Edge of Footing

In reinforced and plain concrete footings, the thickness at the edge shall be not less than 150 mm for footings on soils, nor less than 300 mm above the tops of piles for footings on piles.
34.1.3 In the case of plain concrete pedestals, the angle between the plane passing through the bottom edge of the pedestal and the corresponding junction edge of the column with pedestal and the horizontal plane (see Fig. 20) shall be governed by the expression:

\[ \tan \alpha \leq 0.9 \times \sqrt{\frac{100q_0}{f_{ck}}} + 1 \]

where

\[ q_0 = \text{calculated maximum bearing pressure at the base of the pedestal in N/mm}^2 \]

\[ f_{ck} = \text{characteristic strength of concrete at 28 days in N/mm}^2 \].

34.2 Moments and Forces

34.2.1 In the case of footings on piles, computation for moments and shears may be based on the assumption that the reaction from any pile is concentrated at the centre of the pile.

34.2.2 For the purpose of computing stresses in footings which support a round or octagonal concrete column or pedestal, the face of the column or pedestal shall be taken as the side of a square inscribed within the perimeter of the round or octagonal column or pedestal.

34.2.3 Bending Moment

34.2.3.1 The bending moment at any section shall be determined by passing through the section a vertical plane which extends completely across the footing, and computing the moment of the forces acting over the entire area of the footing on one side of the said plane.

34.2.3.2 The greatest bending moment to be used in the design of an isolated concrete footing which supports a column, pedestal or wall, shall be the moment computed in the manner prescribed in 34.2.3.1 at sections located as follows:

a) At the face of the column, pedestal or wall, for footings supporting a concrete column, pedestal or wall;

b) Halfway between the centre-line and the edge of the wall, for footings under masonry walls; and

c) Halfway between the face of the column or pedestal and the edge of the gusseted base, for footings under gusseted bases.

34.2.4 Shear and Bond

34.2.4.1 The shear strength of footings is governed by the more severe of the following two conditions:

a) The footing acting essentially as a wide beam, with a potential diagonal crack extending in a plane across the entire width; the critical section for this condition shall be assumed as a vertical section located from the face of the column, pedestal or wall at a distance equal to the effective depth of footing for footings on piles.

b) Two-way action of the footing, with potential diagonal cracking along the surface of truncated cone or pyramid around the concentrated load; in this case, the footing shall be designed for shear in accordance with appropriate provisions specified in 31.6.
34.2.4.2 In computing the external shear or any section through a footing supported on piles, the entire reaction from any pile of diameter $D_p$ whose centre is located $D_p/2$ or more outside the section shall be assumed as producing shear on the section; the reaction from any pile whose centre is located $D_p/2$ or more inside the section shall be assumed as producing no shear on the section. For intermediate positions of the pile centre, the portion of the pile reaction to be assumed as producing shear on the section shall be based on straight line interpolation between full value at $D_p/2$ outside the section and zero value at $D_p/2$ inside the section.

34.2.4.3 The critical section for checking the development length in a footing shall be assumed at the same planes as those described for bending moment in 34.2.3 and also at all other vertical planes where abrupt changes of section occur. If reinforcement is curtailed, the anchorage requirements shall be checked in accordance with 26.2.3.

34.3 Tensile Reinforcement

The total tensile reinforcement at any section shall provide a moment of resistance at least equal to the bending moment on the section calculated in accordance with 34.2.3.

34.3.1 Total tensile reinforcement shall be distributed across the corresponding resisting section as given below:

a) In one-way reinforced footing, the reinforcement extending in each direction shall be distributed uniformly across the full width of the footing;

b) In two-way reinforced square footing, the reinforcement extending in each direction shall be distributed uniformly across the full width of the footing; and

c) In two-way reinforced rectangular footing, the reinforcement in the long direction shall be distributed uniformly across the full width of the footing. For reinforcement in the short direction, a central band equal to the width of the footing shall be marked along the length of the footing and portion of the reinforcement determined in accordance with the equation given below shall be uniformly distributed across the central band:

$$\frac{\text{Reinforcement in central band width}}{\text{Total reinforcement in short direction}} = \frac{2}{\beta + 1}$$

where $\beta$ is the ratio of the long side to the short side of the footing. The remainder of the reinforcement shall be uniformly distributed in the outer portions of the footing.

34.4 Transfer of Load at the Base of Column

The compressive stress in concrete at the base of a column or pedestal shall be considered as being transferred by bearing to the top of the supporting pedestal or footing. The bearing pressure on the loaded area shall not exceed the permissible bearing stress in direct compression multiplied by a value equal to

$$\frac{\sqrt{A_1}}{\sqrt{A_2}}$$

but not greater than 2, where $A_1$ = supporting area for bearing of footing, which in sloped or stepped footing may be taken as the area of the lower base of the largest frustum of a pyramid or cone contained wholly within the footing and having for its upper base, the area actually loaded and having side slope of one vertical to two horizontal; and $A_2$ = loaded area at the column base.
34.4.1 Where the permissible bearing stress on the concrete in the supporting or supported member would be exceeded, reinforcement shall be provided for developing the excess force, either by extending the longitudinal bars into the supporting member, or by dowels (see 34.4.3).

34.4.2 Where transfer of force is accomplished by, reinforcement, the development length of the reinforcement shall be sufficient to transfer the compression or tension to the supporting member in accordance with 26.2.

34.4.3 Extended longitudinal reinforcement or dowels of at least 0.5 percent of the cross-sectional area of the supported column or pedestal and a minimum of four bars shall be provided. Where dowels are used, their diameter shall not exceed the diameter of the column bars by more than 3 mm.

34.4.4 Column bars of diameters larger than 36 mm, in compression only can be dowelled at the footings with bars of smaller size of the necessary area. The dowel shall extend into the column, a distance equal to the development length of the column bar and into the footing, a distance equal to the development length of the dowel.

34.5 Nominal Reinforcement

34.5.1 Minimum reinforcement and spacing shall be as per the requirements of solid slab.

34.5.2 The nominal reinforcement for concrete sections of thickness greater than 1 m shall be 360 mm² per metre length in each direction on each face. This provision does not supersede the requirement of minimum tensile reinforcement based on the depth of the section.

6. Numerical Problems

Example 1

Design an isolated footing for an R.C. column of size 230 mm x 230 mm which carries a vertical load of 500 kN. The safe bearing capacity of soil is 200 kN/m². Use M20 concrete and Fe 415 steel.

Solution

Step 1: Size of footing

Load on column = 600 kN

Extra load at 10% of load due to self weight of soil = 60 kN

Hence, total load, P = 660 kN

Required area of footing, \( A = \frac{P}{5Bc} = \frac{660}{200} = 3.3 \text{ m}^2 \)

Assuming a square footing, the side of footing is \( L = B = \sqrt{3.3} = 1.82 \text{ m} \)

Hence, provide a footing of size 1.85 m x 1.85 m

Net upward pressure in soil, \( p = \frac{600}{1.85 \times 1.85} = 175.3 \text{ kN/m}^2 < 200 \text{ kN/m}^2 \) Hence O.K.

Hence, factored upward pressure of soil, \( p_u = 263 \text{ kN/m}^2 \) and, factored load, \( P_u = 900 \text{ kN} \).
Step 2: Two way shear

Assume an uniform overall thickness of footing, D = 450 mm.
Assuming 12 mm diameter bars for main steel, effective thickness of footing ‘d’ is
\[ d = 450 - 50 - 12 = 382 \text{ mm} \]
The critical section for the two way shear or punching shear occurs at a distance of \(d/2\) from the face of the column (See Fig. 6), where a and b are the sides of the column.

Hence, punching area of footing = \((a + d)^2 = (0.23 + 0.382)^2 = 0.375 \text{ m}^2\)
here a = b = side of column
Punching shear force = Factored load – (Factored upward pressure \(\times\) punching area of footing)
\[ = 900 - (263 \times 0.375) \]
\[ = 801.38 \text{ kN} \]
Perimeter of the critical section = \(4 \,(a+d) = 4 \,(230+382) \)
\[ = 2448 \text{ mm} \]
Therefore, nominal shear stress in punching or punching shear stress \(\zeta_p\) is computed as
\[
\zeta_p = \frac{\text{Punching shear force}}{\text{perimeter} \times \text{effective thickness}} = \frac{801.38 \times 1000}{2448 \times 382} = 0.86 \text{ N/mm}^2
\]
Allowable shear stress = \(k_s \cdot \zeta_c\)
where \(\zeta_c = 0.25 \sqrt{f_{ck}} = 0.25 \sqrt{20} \approx 1.12 \text{ N/mm}^2\)
and, \(k_s = (0.5 + \beta_c) = \left(0.5 + \frac{0.23}{0.23}\right) = 1.0 \); Hence, adopt \(k_s=1\)
Thus, Allowable shear stress = \(k_s \cdot \zeta_c = 1 \times 1.12 = 1.12 \text{ N/mm}^2\)

Since the punching shear stress \((0.86 \text{ N/mm}^2)\) is less than the allowable shear stress \((1.12 \text{ N/mm}^2)\), the assumed thickness is sufficient to resist the punching shear force.

Hence, the assumed thickness of footing D = 450 mm is sufficient.
The effective depth for the lower layer of reinforcement, \(d = 450 - 50 - 6 = 396 \text{ mm}\), and
the effective depth for the upper layer of reinforcement, \(d = 450 - 50 - 12 - 6 = 382 \text{ mm}\).
**Step 3: Design for flexure**

The critical section for flexure occurs at the face of the column (Fig. 7).

The projection of footing beyond the column face is treated as a cantilever slab subjected to factored upward pressure of soil.

Factored upward pressure of soil, $p_u = 263 \text{ kN/m}^2$

Projection of footing beyond the column face, $l = (1850 - 230)/2 = 810 \text{ mm}$

Hence, bending moment at the critical section in the footing is

$$M_u = \frac{p_u l^2}{2} = \frac{263 \times 0.81^2}{2} = 86.28 \text{ kN} \cdot \text{m}$$

Width of footing

The area of steel $A_{st}$ can be determined using the following moment of resistance relation for under reinforced condition given in Annex G – 1.1 b of IS 456 :2000.

$$M_u = 0.87 f_y A_{st} d \left[ 1 - \frac{f_y A_{st}}{b d f_{ck}} \right]$$

Considering 1m width of footing,

$$86.28 \times 10^6 = 0.87 \times 415 \times A_{st} \times 382 \left[ 1 - \frac{415 \times A_{st}}{1000 \times 382 \times 20} \right]$$

Solving the above quadratic relation, we get

$A_{st} = 648.42 \text{ mm}^2$ and $17,761.01 \text{ mm}^2$

Selecting the least and feasible value for $A_{st}$, we have

$A_{st} = 648.42 \text{ mm}^2$

The corresponding value of $p_t = 0.17 \%$

Hence from flexure criterion, $p_t = 0.17 \%$

**Step 4: One way shear**

The critical section for one way shear occurs at a distance ‘$d$’ from the face of the column (Fig. 8).
For the cantilever slab, total Shear Force along critical section considering the entire width \( B \) is

\[
V_u = p_t B (l - d) = 263 \times 1.85 \times (0.81 - 0.382) = 208.24 \text{ kN}
\]

The nominal shear stress is given by

\[
\zeta_v = \frac{V_u}{B d} = \frac{208.24 \times 1000}{1850 \times 382} = 0.30 \text{ N/mm}^2
\]

From Table 61 of SP 16, find the \( p_t \) required to have a minimum design shear strength \( \zeta_c = \zeta_v = 0.30 \text{ N/mm}^2 \) with \( f_{ck} = 20 \text{ N/mm}^2 \).

For \( p_t = 0.175 \% \) the design shear strength \( \zeta_c \) is 0.30 N/mm\(^2\) = \( \zeta_v = 0.30 \text{ N/mm}^2 \).

Hence from one way shear criterion, \( p_t = 0.175 \% \)

Comparing \( p_t \) from flexure and one way shear criterion, provide \( p_t = 0.175 \% \) (larger of the two values)

Hence, \( A_{st} = \frac{p_t}{100} b d = \frac{0.175}{100} 1000 \times 382 = 669 \text{ mm}^2 \)

Provide \( \phi \) 12 mm dia bars at 140 mm c/c.

Therefore, \( A_{st} \) provided = 808 mm\(^2\) > \( A_{st} \) required (609 mm\(^2\)). Hence O.K.

**Step 5: Check for development length**

Sufficient development length should be available for the reinforcement from the critical section. Here, the critical section considered for \( L_d \) is that of flexure.

The development length for 12 mm dia bars is given by

\( L_d = 47 \phi = 47 \times 12 = 564 \text{ mm} \).

Providing 60 mm side cover, the total length available from the critical section is

\[
\frac{1}{2} (L - a) - 60 = \frac{1}{2} (1850 - 230) - 60 = 750 \text{ mm} > L_d. \text{ Hence O.K.}
\]

**Step 6: Check for bearing stress**

The load is assumed to disperse from the base of column to the base of footing at rate of 2H : 1V. Hence, the side of the area of dispersion at the bottom of footing = 230 + 2 (2 x 450) = 2030 mm.

Since this is lesser than the side of the footing (i.e., 1850 mm)

\[ A_1 = 1.85 \times 1.85 = 3.4225 \text{ m}^2 \]
The dimension of the column is 230 mm x 230 mm. Hence, \( A_2 = 0.230 \times 0.230 = 0.0529 \, m^2 \)

\[
\frac{A_1}{A_2} = \frac{3.4225}{0.0529} = 8.04 > 2
\]

Hence, limit the value of \( \frac{A_1}{A_2} = 2 \)

\[
\therefore \text{Permissible bearing stress} = 0.45 \, f_{ck} \left( \frac{A_1}{A_2} \right) = 0.45 \times 20 \times 2 = 18 \, N/mm^2
\]

\[
\text{Actual bearing stress} = \frac{\text{Factored load}}{\text{Area at column base}} = \frac{900 \times 1000}{230 \times 230} = 17.01 \, N/mm^2
\]

Since the Actual bearing stress (17.01 N/mm²) is less than the Permissible bearing stress (18 N/mm²), the design for bearing stress is satisfactory.

Appropriate detailing should be shown both in plan and elevation for the footing as per the recommendations given in SP 34.

**Example 2**

Design an isolated footing for an R.C. column of size 300 mm x 300 mm which carries a vertical load of 800 kN together with an uniaxial moment of 40 kN-m. The safe bearing capacity of soil is 250 kN/m². Use M25 concrete and Fe 415 steel.

**Solution**

**Step 1: Size of footing**

Load on column = 800 kN

Extra load at 10% of load due to self weight of soil = 80 kN

Hence, total load, \( P = 880 \, kN \)

Let us provide a square isolated footing, where \( L = B \)

Equating the maximum pressure of the footing to SBC of soil,

\[
\frac{P}{A} + \frac{M}{Z} = \text{SBC}
\]

i.e., \( \frac{880}{B^2} + \frac{40 \times 6}{B^3} = 250 \)

On solving the above equation, and taking the least and feasible value, \( B = 2 \, m \)

Hence, provide a square footing of size 2 m x 2 m

The maximum and minimum soil pressures are given by

\[
\begin{align*}
\rho_{u,\text{max}} &= \frac{800}{2^2} + \frac{40 \times 6}{2^3} = 230 \, kN/m^2 < 250 \, kN/m^2 \, O.K. \\
\rho_{u,\text{min}} &= \frac{800}{2^2} - \frac{40 \times 6}{2^3} = 170 \, kN/m^2 > 0 \, kN/m^2
\end{align*}
\]

Hence, factored upward pressures of soil are,

\( \rho_{u,\text{max}} = 345 \, kN/m^2 \) and \( \rho_{u,\text{min}} = 255 \, kN/m^2 \)
Further, average pressure at the center of the footing is given by

\[ p_{u,\text{avg}} = 300 \, \text{kN/m}^2 \]

and, factored load, \( P_u = 900 \, \text{kN} \), factored uniaxial moment, \( M_u = 60 \, \text{kN-m} \)

**Step 2: Two way shear**

Assume an uniform overall thickness of footing, \( D = 450 \, \text{mm} \)

Assuming 16 mm diameter bars for main steel, effective thickness of footing ‘d’ is

\[ d = 450 - 50 - 16 - 8 = 376 \, \text{mm} \]

The critical section for the two way shear or punching shear occurs at a distance of \( d/2 \) from the face of the column (Fig. 9), where \( a \) and \( b \) are the dimensions of the column.

![Fig. 9 Critical section in two way shear](image)

Hence, punching area of footing = \((a + d)^2 = (0.30 + 0.376)^2 = 0.457 \, \text{m}^2\)

where \( a = b = \) side of column

Punching shear force = Factored load – (Factored average pressure x punching area of footing)

\[ = 1200 - (300 \times 0.457) \]

\[ = 1062.9 \, \text{kN} \]

Perimeter along the critical section = \(4 \, (a+d) = 4 \, (300 + 376) \)

\[ = 2704 \, \text{mm} \]

Therefore, nominal shear stress in punching or punching shear stress \( \zeta_V \) is computed as

\[ \zeta_V = \frac{\text{Punching shear force}}{\text{perimeter} \times \text{effective thickness}} \]

\[ = \frac{1062.9 \times 1000}{2704 \times 376} = 1.05 \, \text{N/mm}^2 \]

Allowable shear stress = \( k_s \cdot \zeta_C \)

where \( \zeta_C = 0.25 \cdot \sqrt{f_{ek}} = 0.25 \cdot \sqrt{25} = 1.25 \, \text{N/mm}^2 \)

and, \( k_s = (0.5 + \beta_c) = \left(0.5 + \frac{0.30}{0.30}\right) = 1.0 \); Hence, adopt \( k_s = 1 \)

Thus, Allowable shear stress = \( k_s \cdot \zeta_C = 1 \times 1.25 = 1.25 \, \text{N/mm}^2 \)

Since the punching shear stress (1.05 N/mm²) is less than the allowable shear stress (1.25 N/mm²), the assumed thickness is sufficient to resist the punching shear force.

Hence, the assumed thickness of footing \( D = 450 \, \text{mm} \) is sufficient.
The effective depth for the lower layer of reinforcement, \( d = 450 - 50 - 8 = 392 \) mm, and the effective depth for the upper layer of reinforcement, \( d = 450 - 50 - 16 - 8 = 376 \) mm.

**Step 3: Design for flexure**

The critical section for flexure occurs at the face of the column (Fig. 10).

![Fig. 10 Critical section for flexure](image)

The projection of footing beyond the column face is treated as a cantilever slab subjected to factored upward pressure of soil.

Factored maximum upward pressure of soil, \( p_{u,\text{max}} = 345 \) kN/m\(^2\)

Factored upward pressure of soil at critical section, \( p_u = 306.75 \) kN/m\(^2\)

Projection of footing beyond the column face, \( l = (2000 - 300)/2 = 850 \) mm

Bending moment at the critical section in the footing is

\[
M_u = \left[ \frac{345 + 306.75}{2} \right] \times \left[ \frac{2 \times 345 + 306.75}{345 + 306.75} \times \frac{0.85}{3} \right]
\]

\( M_u = 119.11 \) kN-m/ m width of footing

The area of steel \( A_{st} \) can be determined using the following moment of resistance relation for under reinforced condition given in Annex G – 1.1 b of IS 456 :2000.

\[
M_u = 0.87 f_y A_{st} d \left[ 1 - \frac{f_y A_{st}}{b d f_{ck}} \right]
\]

Considering 1m width of footing,

\[
119.11 \times 10^6 = 0.87 \times 415 \times A_{st} \times 376 \left[ 1 - \frac{415 \times A_{st}}{1000 \times 376 \times 25} \right]
\]

Solving the quadratic equation,

\( A_{st} = 914.30 \) mm\(^2\) and \( 21,735.76 \) mm\(^2\)

Selecting the least and feasible value, \( A_{st} = 914.30 \) mm\(^2\)
The corresponding value of $p_t = 0.24\%$

Hence from flexure criterion, $p_t = 0.24\%$

**Step 4: One way shear**

The critical section for one way shear occurs at a distance of ‘d’ from the face of the column (Fig. 11).

![Fig. 11 Critical section for one way shear](image)

Factored maximum upward pressure of soil, $p_{u,\text{max}} = 345\, kN/m^2$

Factored upward pressure of soil at critical section, $p_u = 327.1\, kN/m^2$

For the cantilever slab, total Shear Force along critical section considering the entire width B is

$$V_u = \frac{[\text{Total force}] \times [(1 - d) \times B]}{2}$$

$$V_u = \frac{[345 + 327.1]}{2} \times [(0.85 - 0.376) \times 2]$$

$$V_u = 318.58\, kN$$

The nominal shear stress is given by

$$\zeta_v = \frac{V_u}{B \, d} = \frac{318.58 \times 1000}{2000 \times 376} = 0.42\, N/mm^2$$

From 19 of IS 456:2000, find the $p_t$ required to have a minimum design shear strength $\zeta_C = \zeta_V = 0.42\, N/mm^2$ with $f_{ck} = 25\, N/mm^2$.

For $p_t = 0.365\%$ the design shear strength $\zeta_C$ is $0.42\, N/mm^2 = \zeta_V = 0.42 \, N/mm^2$.

Hence from one way shear criterion, $p_t = 0.365\%$

Comparing $p_t$ from flexure and one way shear criterion, provide $p_t = 0.365\%$ (larger of the two values)

Hence, $A_{st} = \frac{p_t}{100} \times b \times d = \frac{0.365}{100} \times 1000 \times 376 = 1372.4\, mm^2$

Provide $\phi$ 16 mm dia bars at 140 mm c/c.

Therefore, $A_{st}$ provided = 1436 mm$^2$ > $A_{st}$ required (1372.4 mm$^2$). Hence O.K.

**Step 5: Check for development length**

Sufficient development length should be available for the reinforcement from the critical section.

Here, the critical section considered for $L_d$ is that of flexure.
The development length for 16 mm dia bars is given by

\[ L_d = 47 \phi = 47 \times 16 = 752 \text{ mm}. \]

Providing 60 mm side cover, the total length available from the critical section is

\[ \frac{1}{2} (L - a) - 60 = \frac{1}{2} (2000 - 300) - 60 = 790 \text{ mm} > L_d \quad \text{Hence O.K.} \]

**Step 6: Check for bearing stress**

The load is assumed to disperse from the base of column to the base of footing at rate of 2H : 1V.

Hence, the side of the area of dispersion at the bottom of footing = 300 + 2 (2 x 450) = 2100 mm.

Since this is lesser than the side of the footing (i.e., 2000 mm),

\[ A_1 = 2 \times 2 = 4 \text{ m}^2 \]

The dimension of the column is 300 mm x 300 mm.

Hence, \( A_2 = 0.30 \times 0.30 = 0.09 \text{ m}^2 \)

\[ \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{4}{0.09}} = 6.67 > 2 \]

Hence, Limit the value of \( \sqrt{\frac{A_1}{A_2}} < 2 \)

\[ \therefore \text{Permissible bearing stress} = 0.45 \cdot f_{ck} \cdot \sqrt{\frac{A_1}{A_2}} \]

\[ = 0.45 \times 25 \times 2 = 22.5 \text{ N/mm}^2 \]

Actual bearing stress = \( \frac{\text{Factored load}}{\text{Area at column base}} = \frac{1200 \times 1000}{300 \times 300} = 13.33 \text{ N/mm}^2 \)

Since the Actual bearing stress (13.33 N/mm²) is less than the Permissible bearing stress (22.5 N/mm²), the design for bearing stress is satisfactory.

Appropriate detailing should be shown both in plan and elevation for the footing as per the recommendations given in SP 34.