5. COMPRESSION MEMBERS

5.1 Introduction

Column, top chords of trusses, diagonals and bracing members are all examples of compression members. Columns are usually thought of as straight compression members whose lengths are considerably greater than their cross-sectional dimensions.

An initially straight strut or column, compressed by gradually increasing equal and opposite axial forces at the ends is considered first. Columns and struts are termed “long” or “short” depending on their proneness to buckling. If the strut is “short”, the applied forces will cause a compressive strain, which results in the shortening of the strut in the direction of the applied forces. Under incremental loading, this shortening continues until the column yields or “squashes”. However, if the strut is “long”, similar axial shortening is observed only at the initial stages of incremental loading. Thereafter, as the applied forces are increased in magnitude, the strut becomes “unstable” and develops a deformation in a direction normal to the loading axis and its axis is no longer straight. (See Fig.5.1). The strut is said to have “buckled”.
Fig 5.1 'short' vs 'long' columns

**Short Columns**

Buckling behaviour is thus characterized by large deformations developed in a direction (or plane) normal to that of the loading that produces it. When the applied loading is increased, the buckling deformation also increases. Buckling occurs mainly in members subjected to compressive forces. If the member has high bending stiffness, its buckling resistance is high. Also, when the member length is increased, the buckling resistance is decreased. Thus the buckling resistance is high when the member is short or "stocky" (i.e. the member has a high bending stiffness and is short) conversely, the buckling resistance is low when the member is long or "slender".

**Long Columns**
Structural steel has high yield strength and ultimate strength compared with other construction materials. Hence compression members made of steel tend to be slender compared with reinforced concrete or prestressed concrete compression members. Buckling is of particular interest while employing slender steel members. Members fabricated from steel plating or sheeting and subjected to compressive stresses also experience local buckling of the plate elements. This chapter introduces buckling in the context of axially compressed struts and identifies the factors governing the buckling behaviour. Both global and local buckling is instability phenomena and should be avoided by an adequate margin of safety.

Traditionally, the design of compression members was based on Euler analysis of ideal columns which gives an upper band to the buckling load. However, practical columns are far from ideal and buckle at much lower loads. The first significant step in the design procedures for such columns was the use of Perry Robertson's curves. Modern codes advocate the use of multiple-column curves for design. Although these design procedures are more accurate in predicting the buckling load of practical columns, Euler's theory helps in the understanding of the behaviour of slender columns and is reviewed in the following sections.
5.10 Summary

In the previous sections, the behaviour of practical columns subjected to axial compressive loading was discussed and the following conclusions were drawn.

- Very short columns subjected to axial compression fail by yielding. Very long columns fail by buckling in the Euler mode.

- Practical columns generally fail by inelastic buckling and do not conform to the assumptions made in Euler theory. They do not normally remain linearly elastic up to failure unless they are very slender.

- Slenderness ratio (l/r) and material yield stress (f_y) are dominant factors affecting the ultimate strengths of axially loaded columns.

- The compressive strengths of practical columns are significantly affected by (i) the initial imperfection (ii) eccentricity of loading (iii) residual stresses and (iv) lack of distinct yield point and strain hardening.Ultimate load tests on practical columns reveal a scatter band of results shown in Fig. 5.19. A lower bound curve of the type shown therein can be employed for design purposes.
5.11 Concluding remarks

The elastic buckling of an ideally straight column pin ended at both ends and subjected to axial compression was considered. The elastic buckling load was shown to be dependent on the slenderness ratio \( (\ell/r) \) of the column. Factors affecting the column strengths (viz. initial imperfection, eccentricity of loading, residual stresses and lack of well-defined elastic limit) were all individually considered. Finally a generalized column strength curve (taking account of all these factors) has been suggested, as the basis of column design curves employed in Design Practices. The concept of “effective length” of the column has been described, which could be used as the basis of design of columns with differing boundary conditions.

The phenomenon of Elastic Torsional and Torsional-flexural buckling of a perfect column were discussed conceptually. The instability effects due to torsional buckling of slender sections are explained and discussed.

Design of columns using multiple column curves as given in the code; was discussed. Built-up fabricated members frequently employed (when rolled sections are found inadequate) were discussed in detail. Design guidance is provided for laced/battened columns. Steps in the design of axially loaded column were listed.
5.2 Elastic buckling of an ideal column or struct with pinned end

Fig 5.2 Buckling of pin-ended column

The classical Euler analysis of the elastic behaviour of an idealized, pin-ended, uniform strut makes the following assumptions.

- The material is homogeneous and linearly elastic (i.e. it obeys Hooke’s Law).
- The strut is perfectly straight and there are no imperfections.
- The loading is applied at the centroid of the cross section at the ends.

We will assume that the member is able to bend about one of the principal axes. (See Fig. 5.2). Initially, the strut will remain straight for all values of \( P \), but at a particular value \( P = P_{cr} \), it buckles. Let the buckling deformation at a section distant \( x \) from the end B be \( y \).

The bending moment at this section = \( P_{cr}y \)

The differential equation governing the deformation can be obtained by considering moment equilibrium about point C as
\[ p_{cr} y + EI \frac{d^2y}{dx^2} = 0 \]  \hspace{1cm} (5.1)

The general solution for this differential equation is given by

\[ y = A_1 \cos x \sqrt{\frac{p_{cr}}{EI}} + B_1 \sin x \sqrt{\frac{p_{cr}}{EI}} \]  \hspace{1cm} (5.2)

Where \( A_1 \) and \( B_1 \) are constants.

Since \( y = 0 \) when \( x = 0 \), \( A_1 = 0 \).

Also \( y = 0 \) when \( x = L \) gives

\[ B_1 \sin L \sqrt{\frac{p_{cr}}{EI}} = 0 \]

Thus, either \( B_1 = 0 \) or

\[ B_1 \sin L \sqrt{\frac{p_{cr}}{EI}} = 0 \]

\( B_1 = 0 \) means \( y = 0 \) for all values of \( x \) (i.e. the column remains straight).

Alternatively

This equation is satisfied only when

\[ L \sqrt{\frac{p_{cr}}{EI}} = 0, \pi, 2\pi, \ldots \]  \hspace{1cm} (5.3)

This gives

\[ p_{cr} = \frac{\pi^2 EI}{L^2}, \frac{4\pi^2 EI}{L^2}, \ldots, \frac{n^2 \pi^2 EI}{L^2} \]

Where \( n \) is any integer.
While there are several buckling modes each corresponding to \( n = 1, 2, 3 \ldots \) (See Fig. 5.3) the lowest stable buckling mode corresponds to \( n = 1 \).

The lowest value of the critical load (i.e. the load causing buckling) is given by

\[
P_{cr} = \frac{\pi^2 EI}{L^2}
\]

Thus the Euler buckling analysis for a “straight” strut; will lead to the following conclusions:

1. The strut can remain straight for any value of \( P \).
2. Under incremental loading, when \( P \) reaches a value of \( P_{cr} = \frac{\pi^2 EI}{L^2} \) the strut can buckle in the shape of a half-sine wave; the amplitude of this buckling deflection is indeterminate.

3. At higher values of the loads given by \( n^2 \pi^2 EI / L^2 \) other sinusoidal buckled shapes (\( n \) half waves) are possible. However, it is possible to show that the column will be in unstable equilibrium for all values of \( P > \frac{\pi^2 EI}{L^2} \) whether it be straight or buckled.

This means that the slightest disturbance will cause the column to deflect away from its
original position. Elastic Instability may be defined in general terms as a condition in which the structure has no tendency to return to its initial position when slightly disturbed, even when the material is assumed to have an infinitely large yield stress. Thus \( P_{cr} = \frac{\pi^2 E I}{L^2} \) represents the maximum load that the strut can usefully support.

It is often convenient to study the onset of elastic buckling in terms of the mean applied compressive stress (rather than the force). The mean compressive stress at buckling, \( f_{cr} \), is given by

\[
\frac{f_{cr}}{A} = \frac{P_{cr}}{AL^2} = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 E}{\lambda^2}
\]

(5.5)

Where \( A \) = area of cross section of the strut, \( r \) = radius of gyration of the cross section, \( (I = Ar^2) \) and \( \lambda = L/r \)

\( \lambda \) = the slenderness ratio of the column defined by

The equation \( f_{cr} = \frac{\pi^2 E}{\lambda^2} \), implies that the critical stress of a column is inversely proportional to the square of the slenderness ratio of the column (see Fig. 5.4).

![Euler buckling relation between \( f_{cr} \) and \( \lambda \)](image_url)
5.3 Strength curve for all ideal strut

We will assume that the stress-strain relationship of the material of the column is as shown in Fig. 5.5. Such a strut under compression can therefore resist only a maximum force given by $f_y A$, when plastic squashing failure would occur by the plastic yielding of the entire cross section; this means that the stress at failure of a column can never exceed $f_y$, shown by $A-A'$ in Fig. 5.6(a).

![Fig 5.5 Idealised elastic-plastic relationship for steel](image)

From Fig. 5.4, it is obvious that the column would fail by buckling at a stress given by $(\pi^2 EL / \lambda^2)$

![Fig 5.6(a) Strength curve for an axially loaded initially straight pin-ended column](image)
This is indicated by B-B¹ in Fig. 5.6(a), which combines the two types of behaviour just described. The two curves intersect at C. Obviously the column will fail when the axial compressive stress equals or exceeds the values defined by ACB. In the region AC, where the slenderness values are low, the column fails by yielding. In the region CB, the failure will be triggered by buckling. The changeover from yielding to buckling failure occurs at the point C, defined by a slenderness ratio given by $\lambda_c$ and is evaluated from

\[
\lambda_c = \pi \sqrt{\frac{E}{f_y}}
\]

Plots of the type Fig. 5.6(a) are sometimes presented in a non-dimensional form as illustrated in Fig. 5.6(b). Here $(f_t / f_y)$ is plotted against a generalized slenderness given by

\[
\lambda_g = \lambda / \lambda_c = \sqrt{\frac{f_y}{f_{cr}}}
\]
This single plot can be employed to define the strength of all axially loaded, initially straight columns irrespective of their $E$ and $f_y$ values. The change over from plastic yield to elastic critical buckling failure occurs when $\lambda_g = 1$ (i.e., when $f_y = \sigma_{cr}$), the corresponding slenderness ratio is $\pi \sqrt{E / f_y}$. This slenderness ratio demarcates short and long columns.
5.4 Strength of compression members in practice

The highly idealized straight form assumed for the struts considered so far cannot be achieved in practice. Members are never perfectly straight and they can never be loaded exactly at the centroid of the cross section. Deviations from the ideal elastic plastic behaviour defined by Fig. 5 are encountered due to strain hardening at high strains and the absence of clearly defined yield point in some steel. Moreover, residual stresses locked-in during the process of rolling also provide an added complexity.

Thus the three components, which contribute to a reduction in the actual strength of columns (compared with the predictions from the “ideal” column curve) are

(i) Initial imperfection or initial bow.
(ii) Eccentricity of application of loads.
(iii) Residual stresses locked into the cross section.

5.4.1 The effect of initial out-of-straightness

![Diagram of pin-ended strut with initial imperfection](image)

**Fig 5.7 Pin-ended strut with initial imperfection**

A pin-ended strut having an initial imperfection and acted upon by a gradually increasing axial load is shown in Fig 5.7. As soon as the load is applied, the member experiences a bending moment at every cross section, which in turn causes a bending
deformation. For simplicity of calculations, it is usual to assume the initial shape of the column defined by

\[ y_0 = a_0 \sin \frac{\pi x}{l} \]  

(5.8)

where \( a_0 \) is the maximum imperfection at the centre, where \( x = l / 2 \). Other initial shapes are, of course, possible, but the half sine-wave assumed above corresponding to the lowest mode shape, represents the greatest influence on the actual behaviour, and hence is adequate.

Provided the material remains elastic, it is possible to show that the applied force, \( P \), enhances the initial deflection at every point along the length of the column by a multiplier factor, given by

\[ MF = \frac{1}{1 - \left( \frac{P}{P_{cr}} \right)} \]  

(5.9)

The deflection will tend to infinity, as \( P \) is tends to \( P_{cr} \) as shown by curve-A, in Fig. 5.8. However the column will fail at a lower load \( P_f \) when the deflection becomes large enough. The corresponding stress is denoted as \( f_f \).
If a large number of imperfect columns are tested to failure, and the data points representing the values of the mean stress at failure plotted against the slenderness ($\lambda$) values, the resulting lower bound curve would be similar to the curve shown in Fig. 5.9.

![Fig 5.9 Strength curves for strut with initial imperfection](image)

For very stocky members, the initial out of straightness – which is more of a function of length than of cross sectional dimensions – has a very negligible effect and the failure is at plastic squash load. For a very slender member, the lower bound curve is close to the elastic critical stress ($f_{cr}$) curve. At intermediate values of slenderness the effect of initial out of straightness is very marked and the lower bound curve is significantly below the $f_y$ line and $f_{cr}$ line.

### 5.4.2 The effect of eccentricity of applied loading

As has already been pointed out, it is impossible to ensure that the load is applied at the exact centroid of the column. Fig. 5.10 shows a straight column with a small eccentricity ($e$) in the applied loading. The applied load ($P$) induces a bending moment ($P.e$) at every cross section. This would cause the column to deflect laterally, in a manner similar to the initially deformed member discussed previously. Once again the greatest compressive stress will occur at the concave face of the column at a
section midway along its length. The load-deflection response for purely elastic and elastic-plastic behaviour is similar to those described in Fig. 5.8 except that the deflection is zero at zero load.

![Diagram of load-deflection response for purely elastic and elastic-plastic behaviour](image)

**Fig 5.10 Strength curves for eccentrically loaded columns**

The form of the lower bound strength curve obtained by allowing for eccentricity is shown in Fig. 5.10. The only difference between this curve and that given in Fig. 5.9 is that the load carrying capacity is reduced (for stocky members) even for low values of $\lambda$.

### 5.4.3 The effect of residual stress

As a consequence of the differential heating and cooling in the rolling and forming processes, there will always be inherent residual stresses. A simple explanation for this phenomenon follows. Consider a billet during the rolling process when it is shaped into an I section. As the hot billet shown in Fig. 5.11(a) is passed successively through a series of rollers, the shapes shown in 5.11(b), (c) and (d) are gradually obtained. The outstands (b-b) cool off earlier, before the thicker inner elements (a-a) cool down.
Fig 5.11 Various stages of rolling a steel grider

As one part of the cross section (b-b) cools off, it tends to shrink first but continues to remain an integral part of the rest of the cross section. Eventually the thicker element (a) also cools off and shrinks. As these elements remain composite with the edge elements, the differential shrinkage induces compression at the outer edges (b). But as the cross section is in equilibrium – these stresses have to be balanced by tensile stresses at inner location (a). These stress called residual stresses, can sometimes be very high and reach upto yield stress.

Fig. 5.12 Distribution of residual stresses

Consider a short compression member (called a “stub column”, having a residual stress distribution as shown in Fig. 5.12. When this cross section is subjected to an applied uniform compressive stress (f_a) the stress distribution across the cross section can be obtained by superposing the applied stress over the residual stress f_r, provided
the total stress nowhere reaches yield, the section continues to deform elastically. Under incremental loading, the flange tips will yield first when \[ (f_a + f_r) = f_y \]. Under further loading, yielding will spread inwards and eventually the web will also yield. When \( f_a = f_y \), the entire section will have yielded and the column will get squashed.

Only in a very stocky column (i.e. one with a very low slenderness) the residual stress causes premature yielding in the manner just described. The mean stress at failure will be \( f_y \), i.e. failure load is not affected by the residual stress. A very slender strut will fail by buckling, i.e. \( f_{cr} \ll f_y \). For struts having intermediate slenderness, the premature yielding at the tips reduces the effective bending stiffness of the column; in this case, the column will buckle elastically at a load below the elastic critical load and the plastic squash load. The column strength curve will thus be as shown in Fig. 5.13.

Notice the difference between the buckling strength and the plastic squash load is most pronounced when

\[
\lambda = \frac{l}{r} = \pi \left(\frac{E}{f_y}\right)^{1/2}
\]

![Fig 5.13 Strength curve for columns having residual stress](image)
5.4.4 The effect of strain-hardening and the absence of clearly defined yield point

If the material of the column shows strain hardening after an yield plateau, the onset of first yield will not be affected, but the collapse load may be increased. Designers tend to ignore the effect of strain hardening which in fact provides an additional margin of safety.

High strength steels generally have stress-strain curves without a clear yield point. At stresses above the limit of proportionality ($f_p$), the material behaviour is non-linear and on unloading and reloading the material is linear-elastic. Most high strength structural steels have an ultimate stress beyond which the curve becomes more or less horizontal. Some steels do not have a plastic plateau and exhibit strain-hardening throughout the inelastic range. In such cases, the yield stress is generally taken as the 0.2% proof stress, for purposes of computation.

5.4.5 The effect of all features taken together

In practice, a loaded column may experience most, if not all, of the effects listed above i.e. out of straightness, eccentricity of loading, residual stresses and lack of clearly defined yield point and strain hardening occurring simultaneously. Only strain hardening tends to raise the column strengths, particularly at low slenderness values. All other effects lower the column strength values for all or part of the slenderness ratio range.

When all the effects are put together, the resulting column strength curve is generally of the form shown in Fig. 5.14. The beneficial effect of strain hardening at low slenderness values is generally more than adequate to provide compensation for any loss of strength due to small, accidental eccentricities in loading. Although the column strength can exceed the value obtained from the yield strength ($f_y$), for purposes of structural design, the column strength curve is generally considered as having a cut off at $f_y$, to avoid large plastic compressive deformation. Since it is impossible to quantify the variations in geometric imperfections, accidental eccentricity, residual stresses and
material properties, it is impossible to calculate with certainty, the greatest reduction in strength they might produce in practice. Thus for design purposes, it may be impossible to draw a true lower bound column strength curve. A commonly employed method is to construct a curve on the basis of specified survival probability. (For example, over 98% of the columns to which the column curve relates, can be expected - on a statistical basis – to survive at applied loads equal to those given by the curve). All design codes provide column curves based on this philosophy. Thus a lower band curve (Fig 5.14) or a family of such curves is used in design.

![Column strength curves for struts used in practice](image)

**Fig. 5.14 Column strength curves for struts used in practice**
5.5 The concepts of effective lengths

So far, the discussion in this chapter has been centred around pin-ended columns. The boundary conditions of a column may, however, be idealized in one of the following ways:

- Both the ends pin jointed (i.e. the case considered before)
- Both ends fixed.
- One end fixed and the other end pinned.
- One end fixed and the other end free.

By setting up the corresponding differential equations, expressions for the critical loads as given below are obtained and the corresponding buckled shapes are given in Fig. 18.

Both ends fixed:

\[ p_{cr} = \frac{4\pi^2 EI}{L^2} = \frac{\pi^2 E}{\left[\left(\frac{L}{2}\right)r\right]^2} \]

One end fixed and the other end pinned:

\[ p_{cr} = \frac{2\pi^2 EI}{L^2} = \frac{\pi^2 E}{\left[\left(L/\sqrt{2}\right)r\right]^2} \]

One end fixed and the other end free:

\[ p_{cr} = \frac{\pi^2 EI}{4L^2} = \frac{\pi^2 E}{\left[\left(2L\right)/r\right]^2} \]
Using the column, pinned at both ends as the basis of comparison, the critical load in all the above cases can be obtained by employing the concept of “effective length”, $L_e$.

It is easily verified that the calculated effective length for the various end conditions are as in Table 5.1

**Table 5.1 Effective length of compression members**

<table>
<thead>
<tr>
<th>Boundary conditions</th>
<th>Theory</th>
<th>Code value (Cl.7.2.2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both ends pin ended</td>
<td>1.0L</td>
<td>1.0L</td>
</tr>
<tr>
<td>Both ends fixed</td>
<td>0.5L</td>
<td>0.65L</td>
</tr>
<tr>
<td>One end fixed and the other end pinned</td>
<td>0.707L</td>
<td>0.8L</td>
</tr>
<tr>
<td>One end fixed, and the other free to sway</td>
<td>1.2L</td>
<td>1.2L</td>
</tr>
<tr>
<td>One end fixed and the other end free</td>
<td>2.0L</td>
<td>2.0L</td>
</tr>
</tbody>
</table>
It can be seen that the effective length corresponds to the distance between the points of inflection in the buckled mode. The effective column length can be defined as the length of an equivalent pin-ended column having the same load-carrying capacity as the member under consideration. The smaller the effective length of a particular column, the smaller its danger of lateral buckling and the greater its load carrying capacity. It must be recognized that column ends in practice are neither perfectly fixed nor perfectly hinged. The designer may have to interpolate between the theoretical values given above, to obtain a sensible approximation to actual restraint conditions. Effective lengths prescribed by the code are also given in Table 5.1.

5.5.1 Effective lengths in different planes

![Figure 5.16 Columns with different effective length L](image)

The restraint against buckling may be different for buckling about the two column axes. For example, if a column of solid rectangular section were to be connected to the support with a single bolt at either end, it will be like a hinged-hinged column with $L_e$ equal to the distance between the bolts. However, in the perpendicular plane, the column cannot rotate without bending the bolts and will be liked a fixed-fixed column with $L_e$ equal to half the distance between the bolts. Fig 5.16(a) shows a pin-ended column of I section braced about the minor axis against lateral movement (but not...
rotationally restrained) at spacing L / 3. The minor axis buckling mode would be with an effective pin-ended column length \((Le)_y\) of L / 3. If there was no major axis bracing the effective length for buckling about the major axis \((Le)_x\) would remain as L. Therefore, the design slenderness about the major and minor axis would be \(L / r_x\) and \((L / 3)r_y\), respectively. Generally \(r_x < 3r_y\) for all I sections, hence the major axis slenderness \((L / r_x)\) would be greater, giving the lower value of critical load, and failure would occur by major axis buckling. Anyway, checks should be carried out about both the axes.

5.5.2 Effective lengths of columns in frames

Fig 5.17 Limited frames and corresponding effective length charts of IS: 800(draft)

(a) Limited frame and (b) effective length ratio \((K3 = \infty)\), for non-sway frames. (c) Limited frames and (d) effective length ratios (without partial bracing, \(K3 = 0\)), for sway frames
For compression members in rigid-jointed frames the effective length is directly related to the restraint provided by all the surrounding members. In a frame the interaction of all the members occurs because of the frame buckling as a whole rather than column buckling. For individually design purposes, the behaviour of a limited region of the frame is considered. The limited frame comprises the column under consideration and each immediately adjacent member treated as if it were fixed at the far end. The effective length of the critical column is then obtained from a chart which is entered with two coefficients $k_1$, and $k_2$, the values of which depends upon the stiffnesses of the surrounding members $k_u$, $k_{TL}$ etc. Two different cases are considered viz. columns in non-sway frames and columns in sway frames. All these cases as well as effective length charts are shown in Fig.5.17. For the non-sway columns, the effective lengths will vary from 0.5 to 1.0 depending on the values of $k_1$ and $k_2$, while for the sway columns, the variation will be between 1.0 and $\alpha$. These end points correspond to cases of: (1) rotationally fixed ends with no sway and rotationally free ends with no sway; (2) rotationally fixed ends with free sway and rotationally free ends with free sway. The equations for calculating $k_1$, and $k_2$, are given in the code (Cl.7.2.2).
5.6 Torsional and torsional-flexural buckling of columns

**Fig 5.18(a) Plate with unsupported edges**

Folded plate twist click here

**Fig 5.18 (b) Folded plate twist under axial load**
We have so far considered the flexural buckling of a column in which the member deforms by bending in the plane of one of the principal axes. The same form of buckling will be seen in an initially flat wide plate, loaded along its two ends, the two remaining edges being unrestrained. [See Fig. 5.18 (a)]

On the other hand, if the plate is folded at right angles along the vertical centre-line, the resulting angle section has a significantly enhanced bending stiffness. Under a uniform axial compression, the two unsupported edges tend to wave in the Euler type buckles. At the fold, the amplitude of the buckle is virtually zero. A horizontal cross-section at mid height of the strut shows that the cross-section rotates relative to the ends. This mode of buckling is essentially torsional in nature and is initiated by the lack of support at the free longitudinal edges. This case illustrates buckling in torsion, due to the low resistance to twisting (polar moment of inertia) of the member.

Thus the column curves of the type discussed before are only satisfactory for predicting the mean stress at collapse, when the strut buckles by bending in a plane of symmetry of the cross section, referred to as “flexural buckling”. Members with low torsional stiffness (eg. angles, tees etc made of thin walled members) will undergo torsional buckling before flexural buckling. Cruciform sections are generally prone to torsional buckling before flexural buckling. Singly symmetric or un-symmetric cross sections may undergo combined twisting about the shear centre and a translation of the shear centre. This is known as “torsional – flexural buckling”.

Thus a singly symmetric section such as an equal angle or a channel can buckle either by flexure in the plane of symmetry or by a combination of flexure and torsion. All centrally loaded columns have three distinct buckling loads, at least one of which corresponds to torsional or torsional - flexural mode in a doubly symmetric section. Flexural buckling load about the weak axis is almost always the lowest. Hence, we
disregard the torsional buckling load in doubly symmetric sections. In non-symmetric sections, buckling will be always in torsional – flexural mode regardless of its shape and dimensions. However, non-symmetric sections are rarely used.

Thin-walled open sections, such as angles and channels, can buckle by bending or by a combination of bending and twisting. Which of these two modes is critical depends on the shape and dimensions of the cross-section. Hence, torsional-flexural buckling must be considered in their design. This is normally done by calculating an equivalent slenderness ratio and using the same column strength curve as for flexural buckling.
5.7 Design strength

Based on the studies of Ayrton & Perry (1886), the compressive strength of the column can be obtained from the following equation.

\[
(f_y - f_c)(f_c - f_e) = \eta f_c f_e
\]

(5.10)

Where, \(f_y\) = yield stress, \(f_c\) = compressive strength, \(f_e\) = Euler buckling stress, \(\lambda\) = Slenderness ratio \((l/r)\) and \(\eta\) = a parameter allowing for the effects of lack of straightness and eccentricity of loading and can be expressed as \(a\lambda\) where \(a\) is a function of the shape of the cross section. Since Robertson evaluated the mean values of \(a\) for many sections, the design method is termed "Perry-Robertson method".

Equation (5.8) will result in column strength values lower than \(f_y\) even in very low slenderness cases as indicated by the Robertson’s curve in Fig. 5.19. By modifying the slenderness, \(\lambda\) to \((\lambda - \lambda_o)\), a plateau to the design curve can be introduced for low slenderness values. This has the effect of shifting the curve to the right by a value equal to \(\lambda_o\). The value of \(\lambda_o\) may be taken as \(0.2(\pi \sqrt{E/f_y})\). Thus, the elastic critical stress can be calculated as \(f_e = \pi^2 E/(\lambda - \lambda_o)^2\). Note that calculations for \(f_e\) is not needed when \(\lambda \leq \lambda_e\) as the column would fail by squashing at \(f_y\).

![Fig 5.19 Column strength curves](image-url)
5.7.1 Design Strength as per the Code

Common hot rolled and built-up steel members, used for carrying axial compression, usually fail by flexural buckling. The buckling strength of these members is affected by residual stresses, initial bow and accidental eccentricities of load. To account for all these factors, strength of members subjected to axial compression are given by multiple design curves corresponding to buckling class a, b, c, or d, as given below. The design compressive strength of a member is given by (Cl.7.1)

\[ P_d = A_e \cdot f_{cd} \quad (5.11) \]

Where, \( A_e \) = effective sectional area and
\( f_{cd} \) = design stress in compression, obtained as per the following equation:

\[ f_{cd} = \frac{f_y / \gamma_{m0}}{\phi + \left[ \phi^2 - \frac{\lambda^2}{m} \right]^{0.3}} \times \frac{f_y / \gamma_{m0}}{f_y / \gamma_{m0}} \quad (5.12) \]

Where
\[ \phi = 0.5[1+\alpha(\lambda - 0.2) + \lambda^2] \]
\[ \neq f = 0.5[1=\alpha(1-0.2) + 1^2] \]
\[ \lambda_* = \text{non-dimensional effective slenderness ratio} \]
\[ = \sqrt{f_y / f_u} = \sqrt{\frac{f_y (L_e / r)}{\pi^2 E}} \quad (5.13) \]

\( f_{cc} \) = Euler buckling stress = \( \pi^2 E / (KL/r)^2 \)

\( L_e / r \) = effective slenderness ratio, ratio of effective length \( L_e \), to appropriate radius of gyration, \( r \)
\( \alpha \) = imperfection factor given in Table 5.2
\( \chi \) = stress reduction factor
\( c \) = stress reduction factor
\( l_{m0} \neq (\gamma_{m0}) \)
\( \lambda_{\text{mo}} = \) partial safety factor for material strength = 1.1

**Table 5.2 Imperfection factor, \( \alpha \)**

<table>
<thead>
<tr>
<th>Buckling Class</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.21</td>
<td>0.34</td>
<td>0.49</td>
<td>0.76</td>
</tr>
</tbody>
</table>

The classifications of different sections under different buckling class a, b, c or d, is given in Table 5.3 of the Code. Note that thicker sections and welded sections which are likely to have more residual stresses are assigned lower buckling classes. The curves corresponding to different buckling class are presented in non-dimensional form, in Fig 5.20. The selection of an appropriate curve is based on cross section and suggested curves are listed in Table 5.3. Although both hot rolled sections and welded sections have lock-in residual stresses, the distribution and magnitude differ significantly. Residual stresses due to welding are very high and can be of greater consequence in reducing the ultimate capacity of compression members.

![Fig. 5.20 Column Buckling Curves](image-url)
### Table 5.3 Buckling class of cross sections

(Section 7.1.2.2)

<table>
<thead>
<tr>
<th>Cross Section</th>
<th>Limits</th>
<th>Buckling about axis</th>
<th>Buckling Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled I-Sections</td>
<td>$h/b &gt; 1.2 : t_f \leq 40 \text{ mm}$</td>
<td>z-z</td>
<td>a</td>
</tr>
<tr>
<td></td>
<td>$40 \text{ mm} &lt; t_f \leq 100 \text{ mm}$</td>
<td>y-y</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td>$h/b \leq 1.2 : t_f \leq 100 \text{ mm}$</td>
<td>z-z</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td>$t_f &gt; 100 \text{ mm}$</td>
<td>z-z</td>
<td>d</td>
</tr>
<tr>
<td>Welded I-Section</td>
<td>$t_f \leq 40 \text{ mm}$</td>
<td>z-z</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td>$t_f &gt; 40 \text{ mm}$</td>
<td>y-y</td>
<td>c</td>
</tr>
<tr>
<td>Hollow Section</td>
<td>Hot rolled</td>
<td>Any</td>
<td>a</td>
</tr>
<tr>
<td></td>
<td>Cold formed</td>
<td>Any</td>
<td>b</td>
</tr>
<tr>
<td>Welded Box Section</td>
<td>Generally (Except as below)</td>
<td>Any</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td>Thick welds and $b/t_f, z/z &lt; 30$</td>
<td>y-y</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>$h/t_w &lt; 30$</td>
<td>y-y</td>
<td>c</td>
</tr>
<tr>
<td>Channel, Angle, T and Solid Sections</td>
<td>Any</td>
<td>Any</td>
<td>c</td>
</tr>
</tbody>
</table>
5.7.2 Design of angle compression members

When angles are loaded in compression through their centroid, they can be designed as per the procedure described above using curve c. However, angles are usually loaded eccentrically by connecting one of its legs either to a gusset or to an adjacent member. Such angles will buckle in flexural-torsional mode in which there will be significant twisting of the member. Such twisting may be facilitated by the flexibility of the gusset plate and the other members connected to it. To simplify the design, the code considers only two cases – gusset fixed and gusset hinged. The other parameter which will influence the strength of the angle strut is its width-thickness ratio of either leg. Thus, to account for the reduction in strength due to flexural-torsional mode, the code gives an equivalent slenderness ratio as a function of the overall slenderness ratio and the width-thickness ratio. In general, the equivalent slenderness ratio will be less than or equal to the slenderness ratio for flexural buckling $\lambda_{vv}$.

The flexural torsional buckling strength of single angle loaded in compression through one of its legs may be evaluated using the equivalent slenderness ratio, $\lambda_{eq}$, as given below (Cl. 7.5.1.2)

$$\lambda_{eq} = \sqrt{k_1 + k_2 \lambda_w^2 + k_3 \lambda_\phi^2} \quad (5.14)$$

Where

$k_1$, $k_2$, $k_3$ = constants depending upon the end condition, as given in Table 5.4,
\[
\lambda_w = \left( \frac{1}{r_{w}} \right) \text{ and } \lambda_{\phi} = \left( \frac{b_1 + b_2}{\varepsilon \sqrt{\frac{\pi^2 E}{250}}} \right) \text{ (5.15 a,b)}
\]

Where

\( l \) = centre to centre length of the supporting member

\( r_{w} \) = radius of gyration about the minor axis

\( b_1, b_2 \) = width of the two legs of the angle

\( t \) = thickness of the leg

\( \varepsilon \) = yield stress ratio \((250/f_y)^{0.5}\)

<table>
<thead>
<tr>
<th>No. of bolts at the each end connection</th>
<th>Gusset/Connecting member Fixity†</th>
<th>( k_1 )</th>
<th>( k_2 )</th>
<th>( k_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \geq 2 )</td>
<td>Fixed</td>
<td>0.20</td>
<td>0.35</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Hinged</td>
<td>0.70</td>
<td>0.60</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>Fixed</td>
<td>0.75</td>
<td>0.35</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Hinged</td>
<td>1.25</td>
<td>0.50</td>
<td>60</td>
</tr>
</tbody>
</table>

*Stiffeness of in-plane rotational restraint provided to the gusset/connecting member. For partial restraint, the \( \lambda_{eq} \) can be interpolated between the \( \lambda_{eq} \) results for fixed and hinged cases.*

For double angle discontinuous struts, connected back to back, on opposite sides of the gusset or a section, by not less than two bolts or rivets in line along the angles at each end, or by the equivalent in welding, the load may be regarded as applied axially (Cl. 7.5.2). The effective length, \( L_e \), in the plane of end gusset can be taken as between 0.7 and 0.85 times the distance between intersections, depending on the degree of the restraint provided. The effective length, \( L_e \), in the plane perpendicular to that of the end gusset, shall be taken as equal to the distance between centers of intersections.
5.8 Types of column sections

5.8.1 Rolled Steel Sections

Some of the sections employed as compression members are shown in Fig. 5.21. Single angles [Fig 5.21(a)] are satisfactory for bracings and for light trusses. Top chord members of roof trusses are usually made up of double angles back-to-back [Fig 5.21(b)]. The pair of angles used, has to be connected together, so they will act as one unit. Welds may be used at intervals – with a spacer bar between the connecting legs. Alternately “stitch bolts”, washers and “ring fills” are placed between the angles to keep them at the proper distance apart (e.g. to enable a gusset to be connected). Such connections are called tack connections and the terms tack welding or tacks bolting are used.

When welded roof trusses are required, there is no need for gusset plates and T sections [Fig 5.21(c)] can be employed as compression members.

Single channels or C-sections [Fig. 5.21(d)] are generally not satisfactory for use in compression, because of the low value of radius of gyration in the weak direction. They can be used if they could be supported in a suitable way in the weak direction.

Circular hollow sections [Fig. 5.21(e)] are perhaps the most efficient as they have equal values of radius of gyration about every axis. But connecting them is difficult but satisfactory methods have been evolved in recent years for their use in tall buildings.

The next best in terms of structural efficiency will be the square hollow sections (SHS) and rectangular hollow sections, [Fig. 5.21(f)] both of which are increasingly becoming popular in tall buildings, as they are easily fabricated and erected. Welded tubes of circular, rectangular or square sections are very satisfactory for use as columns in a long series of windows and as short columns in walkways and covered
warehouses. For many structural applications the weight of hollow sections required would be only 50% of that required for open profiles like I or C sections.

When the available sections are not suitable, a suitable section may be built-up either by welding or by lacing or battening two sections separated by a suitable distance.

![Cross Section Shapes for Rolled Steel Compression Members](image)

**Fig 5.21: Cross Section Shapes for Rolled Steel Compression Members**

### 5.8.2 Built-up column or fabricated compression members

Compression members composed of two angles, channels, or tees back-to-back in contact or separated by a small distance shall be connected together by tack riveting, tack bolting or tack welding so that the individual sections do not buckle between the tacks before the whole member buckles (Cl. 7.8). Special types of columns called Laced and Battened columns are discussed later in this chapter.

When compression members are required for large structures like bridges, it will be necessary to use built-up sections. They are particularly useful when loads are heavy and members are long (e.g. top chords of Bridge Trusses). Built up sections
[illustrated in Fig. 5.22(a) and 5.22(b)] are popular in India when heavy loads are encountered. The cross section consists of two channel sections connected on their open sides with some type of lacing or latticing (dotted lines) to hold the parts together and ensure that they act together as one unit. The ends of these members are connected with “batten plates” which tie the ends together. Box sections of the type shown in Fig. 5.22(a) or 5.22(b) are sometimes connected by such solid plates either at intervals (battened) or continuously along the length.

A pair of channels connected by cover plates on one side and latticing on the other is sometimes used as top chords of bridge trusses. The gussets at joints can be conveniently connected to the inside of the channels. Plated I sections or built-up I sections are used when the available rolled I sections do not have sufficient strengths to resist column loads [Fig 5.22(c)]. Flange plates or channels may be used in combination with rolled sections to enhance the load resistance of the commonly available sections, which are directly welded or bolted to each other. The lateral dimension of the column is generally chosen at around 1/10 to 1/15 of the height of the column. For purposes of detailing the connection between the flange cover plates or the outer rolled sections to the flanges of the main rolled section, it is customary to design the fasteners for a transverse shear force equal to 2.5% of the compressive load of the column. Columns with open webs may be classified as laced columns or battened columns. In Fig. 5.23, the two channel sections of the column are connected together by batten plates or laces which are shown by dotted lines. A typical lacing or batten plate is shown in Fig. 5.23

![Fig 5.22: Cross Section Shapes for Built-up or fabricated Compression Members](image)
The Code gives simple guidelines for the design of laced (Cl.7.6) and battened columns (Cl.7.7). One of the guidelines is that such columns should where practicable, have a radius of gyration about the axis perpendicular to the plane of lacing not less than the radius of gyration about the axis parallel to the plane of lacing (Cl. 7.6.1.1). To account for the inherent flexibility of laced and battened columns, the Code suggests that the effective slenderness ratio be taken as 5 and 10% respectively more than the calculated values (Cl.7.7.1.4). All columns should be tied at the ends by tie plates or end battens to ensure a satisfactory performance.

The two channel constituents of a laced column, shown in Fig. 5.23(a) and 5.23(b) have a tendency to buckle independently. Lacing provides a tying force to ensure that the channels do not do so. The load that these tying forces cause is generally assumed to cause a shearing force equal to 2.5% of axial load on the column (Cl. 7.6.2.1). (Additionally if the columns are subjected to moments or lateral loading the lacing should be designed for the additional bending moment and shear). To prevent local buckling of unsupported lengths between the two constituent lattice points (or between two battens), the slenderness ratio of individual components should be less than 50 or 0.7 of the slenderness ratio of the built up column (whichever is less).
In laced columns, the lacing should be symmetrical in any two opposing faces to avoid torsion. Lacings and battens are not combined in the same column. The inclination of lacing bars from the axis of the column should not be less than 40° nor more than 70°. (Cl. 7.6.5) The slenderness ratio of the lacing bars should not exceed 145 (Cl. 7.6.3). The effective length of lacing bars is the length between bolts for single lacing and 0.7 of this length for double lacing. The width of the lacing bar should be at least 3 times the diameter of the bolt (Cl. 7.6.3). Thickness of lacing bars should be at least 1/40th of the length between bolts for single lacing and 1/60 of this length for double lacing (both for welded and bolted connections) (Cl. 7.6.4).

In batten columns, the Battens plates at their ends shall be riveted or welded to the main components so as to resist simultaneously a shear \( V_b = \frac{V_t}{C/N} S \) along the column axis and a moment \( M = \frac{V_t}{C/2N} \) at each connection (Cl.7.7). where, \( V_t = \) the transverse shear force; \( C = \) the distance between centre-to-centre of battens, longitudinally; \( N = \) the number of parallel planes of battens (usually 2); \( S = \) the minimum transverse distance between the centroids of the bolt group/welding connecting the batten to the main member.

When plates are used for battens, the end battens shall have an effective depth not less than the perpendicular distance between the centroids of the main members. The intermediate battens shall have an effective depth of not less than three quarters of this distance, but in no case shall the effective depth of any batten be less than twice the width of one member, in the plane of the battens. The thickness of batten or the tie plates shall be not less than one fiftieth of the distance between the innermost connecting lines of rivets or welds, perpendicular to the main member.
Fig. 5.24 Battened Column
5.9 Steps in the design of axially loaded columns

The procedure for the design of an axially compressed column is as follows:

(i) Assume a suitable trial section and classify the section in accordance with the classification in chapter.

(ii) Arrive at the effective length of the column by suitably considering the end conditions.

(iii) Calculate the slenderness ratios ($\lambda$ values) in both minor and major axes direction and also calculate $\lambda_0$ using the formula given below:

$$\lambda_0 = 0.2\pi \sqrt{\frac{E}{f_y}}$$

(iv) Calculate $f_{cd}$ values along both major and minor axes from equation 12

(v) Compute the load that the compression member can resist ($p_d = A_c f_{cd}$)

(vi) Calculate the factored applied load and check whether the column is safe against the given loading. The most economical but safe section can be arrived at by trial and error, i.e. repeating the above process.

The following values are suggested for initial choice of members:

(i) Single angle size: 1/30 of the length of the strut ($L / r \sim 150$)

(ii) Double angle size: 1/35 of the length of strut ($L / r \sim 100-120$)

(iii) Circular hollow sections diameter = 1/40 length ($L / r \sim 100$)